Spinning Charged Ring Model of Electron Yielding Anomalous Magnetic Moment

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Summary: A uniformly charged spinning ring is proposed as a model for the electron. Four parameters, the radius of the ring R, the half-thickness r, the total charge e, and the tangential velocity c are chosen to yield the four electron characteristics, the mass m, the charge e, the spin $\hbar/2$, and the magnetic moment μ_e. The model is completely stable under electromagnetic forces alone. The twice classical value for the gyromagnetic ratio is explained. The size of the electron equals the rationalized Compton wavelength, and the frequency of rotation equals the Compton frequency. The model yields to a higher order approximation the anomalous magnetic moment in agreement with observation.

INTRODUCTION

Ever since discovery of the electron in the nineteenth century, a major goal of physics has been to find an electron model that accounts for the physical properties observed and measured with ever increasing precision. Early models proposed by Abraham[1] and Lorentz[2] provided a physical picture of the electron but could not satisfactorily account for some electrical properties. Later models such as the Dirac point model[3] and the quantum mechanical model have been conformed to experimental data; yet, these mathematical models have only tenuous links to a mechanical and physical structure that is the pursuit of physics (the study of the *physical* nature and properties of the universe). The need for a better electron model is stated by Ivan Sellin who wrote in 1982 that

> "...a good theory of electron structure still is lacking.... There is still no generally accepted explanation for why electrons do not explode under the tremendous Coulomb repulsion forces in an object of small size. Estimates of the amount of energy required to 'assemble' an electron are very large indeed. Electron structure is an unsolved mystery, but so is the structure of most other elementary objects in nature, such as protons [and] neutrons...."[4]

Early models of the electron[1,2] were not realistic, primarily because they did not take into account the spin and magnetic moment of the electron. All of the models proposed so far have had to assume ad hoc non-electromagnetic forces to hold the model together. The model proposed here is the first model ever proposed that is completely stable under electromagnetic forces alone; the model does not radiate.*

THE MODEL AND SOME OF ITS CONSEQUENCES

An electron is assumed to be a uniform surface charge density σ that forms a ring of radius R and half-thickness r spinning about the axis of symmetry with the angular velocity $\omega = c/R$, as

* Recently, we learned that electromagnetic ring models have been proposed by Parson, Compton, Iida, Jennison, and Bostick. References to these proposals can be found in other papers available from Common Sense Science.

shown in Figure 1. The uniform surface charge density over the ring σ must be chosen to yield the total charge e; thus,

$$e = A\sigma = 4\pi^2 \sigma r R \tag{1}$$

where $A = (2\pi r)(2\pi R)$ is the surface area when $r \ll R$.

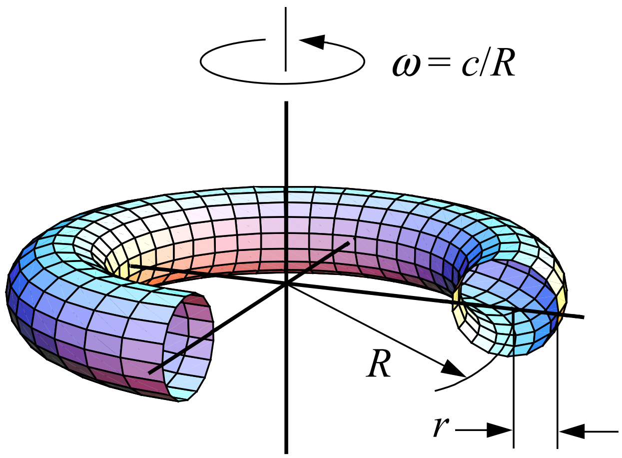

Figure 1: The Spinning Ring Model of the Electron

The mass of the electron is obtained from the classical electromagnetic energy of the charged spinning ring and mass-energy equivalence. The total electromagnetic energy of the spinning ring E_t is given by the electrostatic energy,

$$E_e = \frac{e^2}{2C} \tag{2}$$

where C is the capacitance of the spinning ring, plus the magnetostatic energy,

$$E_m = \frac{LI^2}{2} \tag{3}$$

where L is the self-inductance of the spinning ring, and the current I is

$$I = \frac{e\omega}{2\pi} = \frac{ec}{2\pi R} \tag{4}$$

For the thin ring where $r \ll R$ it may be shown[5,6,7] that the capacitance and inductance have the values

$$C = \frac{4\pi^2 \varepsilon_o R}{\ln(8R/r)} \tag{5}$$

$$L = \mu_o R\left[\ln(8R/r) - 2\right] \approx \mu_o R\left[\ln(8R/r)\right] \tag{6}$$

Equations (2) through (6) yield equipartition of electric and magnetic energies such that

$$\frac{mc^2}{2} = \frac{E_t}{2} \approx E_m \approx E_e = \frac{e^2}{8\pi^2\,\varepsilon_o\,R}\ln(8R/r) \tag{7}$$

The model does not radiate as the sources, the surface charge density σ and the surface current density $\sigma c/2\pi R$, are constant and do not change with time. When "charges occupy the entire circle continuously, then...the total radiation field will vanish...; this implies that a continuous ring current will not radiate...."[8]

The tangential speed c of the spinning ring does not violate any principle. The ring is *not* a material ring, so no matter or mass is traveling with the speed c. Actually the velocity may be regarded as necessarily c, because the model may be viewed as consisting of electric and magnetic fields only; and electromagnetic fields necessarily propagate with the velocity c in a vacuum.

STABILITY OF THE MODEL AS A FUNCTION OF THE HALF-THICKNESS OF THE RING

The essential weakness of prior models of the electron has been that forces of unknown origin have to be postulated ad hoc to hold the electron together against electrostatic repulsion. In contrast, the present model is completely stable under the action of classical electromagnetic forces alone, and no forces of unknown origin have to be postulated.

The magnetic force on the thickness of the ring produces an inward directed pressure P_m or "pinch" force, $\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$; thus,

$$P_m = -\sigma v B \tag{8}$$

where B is the magnetic field at the surface of the thickness due to the spinning of the charged ring. Here the surface charge σ moves with the velocity $v = c$, so the pinch pressure becomes

$$P_m = -\sigma c B \tag{9}$$

Integrating the line integral of **B** around the thickness yields, according to the inappropriately named "Ampere integral law,"

$$2\pi r B = \mu_o I \tag{10}$$

Using equation (4), the magnetic field at the thickness becomes

$$B = \frac{e}{4\pi^2\varepsilon_o c r R} \tag{11}$$

From equations (1), (9), and (11) the magnetostatic pressure on the thickness is then given by

$$P_m = -\frac{e^2}{16\pi^4\varepsilon_o r^2 R^2} = -\frac{B^2}{\mu_o} \tag{12}$$

The latter result, varying as B^2/μ_o is the well known inward directed magnetostatic stress of the field.

The electrostatic repulsive force $\mathbf{F} = q\mathbf{E}$ on the thickness of the ring produces an outward pressure or tension P_e given by

$$P_e = +\sigma E \tag{13}$$

where E is the electric field at the surface of the thickness. From Gauss's law and equation (1), the electric field at the surface of the thickness is

$$E = \frac{\sigma}{\varepsilon_o} = \frac{e}{4\pi^2\, \varepsilon_o r R} = \varepsilon_o E^2 \tag{14}$$

Combining equations (13) and (14), the electrostatic outward directed tension or pressure becomes

$$P_e = \frac{e^2}{16\pi^4 \varepsilon_o r^2 R^2} = \varepsilon_o E^2 \tag{15}$$

The latter result, varying as $\varepsilon_o E^2$ is the well known electrostatic stress or tension of the field.

Comparing equations (12) and (15), it is seen that the outward electrostatic repulsive pressure is balanced exactly by the inward magnetostatic pressure or pinch; thus,

$$P_e = -P_m \tag{16}$$

The thickness of the ring thus holds itself together in equilibrium under classical electrical and magnetic forces alone.

STABILITY OF THE MODEL AS A FUNCTION OF THE RADIUS OF THE RING

In addition to the stability of the thickness involving r, the ring as a closed current loop of radius R is also stable under classical electric and magnetic forces alone. To discuss the stability of the ring as a function of R, it is necessary to use the fundamental Weber[9] electrodynamics as generalized by Wesley,[10] which (unlike the Maxwell theory) prescribes the force between two moving charges from the outset. In particular, the force between two charges q at $\mathbf{s}$ and q' at $\mathbf{s}'$, separated by the distance $\mathbf{S} = \mathbf{s} - \mathbf{s}'$ with relative velocity $\mathbf{V} = \mathbf{v} - \mathbf{v}'$ is

$$4\pi\varepsilon_o c^2\, \mathbf{F} = \frac{qq'\mathbf{S}}{S^3}\left[c^2 + V^2 - \frac{3(\mathbf{V}\cdot\mathbf{S})^2}{2S^2} + \mathbf{S}\cdot\frac{d\,\mathbf{V}}{dt}\right] \tag{17}$$

The outward force f_e per unit length of the ring $R\,d\phi$ due to electrostatic repulsion (the first term in the bracket on the right of equation (17)) may be obtained by considering the change in electrostatic energy when the ring is enlarged from R to $R+dR$. From equations (2) and (7), this electrostatic repulsion per unit length becomes

$$f_e = \frac{\partial\left[q^2/2C\right]}{\partial R} \approx \left(\frac{e^2}{8\pi^2\varepsilon_o}\right)\frac{\ln(8\,R/r)}{R^2} \tag{18}$$

Similarly the force f_m per unit length of the ring of radius R due to magnetostatic forces may be obtained by considering the change in magnetostatic energy for an expansion of the ring from R to $R+dR$. From equations (3) and (7) this magnetostatic repulsive force per unit length becomes

$$f_m = \frac{\partial\left(LI^2/2\right)}{\partial R} \approx \left(\frac{e^2}{8\pi^2\varepsilon_o}\right)\frac{\ln\left(8R/r\right)}{R^2} = f_e \tag{19}$$

The fact that the force f_m is outward may be seen from the fact that the pinch force on the thickness is slightly greater on the inside of the thickness than on the outside of the thickness, the magnetic field being slightly stronger inside then outside. From equations (18) and (19), the net outward force on the ring per unit length due to the electrostatic and magnetostatic effects is

$$f = f_e + f_m \approx \frac{e^2 \ln(8R/r)}{4\pi^2 \varepsilon_o R^2} \tag{20}$$

These forces arise from the first, second and third terms in the bracket on the right side of equation (17).

The effect of the last term in the bracket on the right of equation (17) may be obtained by noting that for $r = 0$ (an approximation that is justified, as $r \ll R$) the model gives*

$$\mathbf{S} \cdot d\mathbf{V}/dt = (\mathbf{s} - \mathbf{s}') \cdot \left(\frac{d\mathbf{v}}{dt} - \frac{d\mathbf{v}'}{dt} \right) = -2R^2\omega^2 \left[1 - \cos(\phi - \phi') \right] \tag{21}$$

where ϕ is the angular position of q and ϕ' is the angular position of q' and both charges are fixed at the same radial distance R with the same angular velocity $\omega = \dot{\phi} = \dot{\phi}'$. The situation is symmetric about the axis of symmetry of the ring, so averaging the angle $(\phi - \phi')$ over the ring yields zero for the cosine part of equation (21); and

$$\left\langle \mathbf{S} \cdot d\mathbf{V}/dt \right\rangle_\phi = -2R^2\omega^2 - 2c^2 \tag{22}$$

Substituting this result (22) into equation (17) shows that the effect of the acceleration term is just twice that of the electrostatic force and is in the opposite direction. Thus the acceleration force per unit length f_a is directed inward and is given by

$$f_a = -2 f_e \tag{23}$$

From the equality of the electrostatic and magnetostatic forces, it may be seen that the total force of electromagnetic origin on the ring from equations (20) and (23) is zero; thus,

$$f_a + f_e + f_m = 0 \tag{24}$$

The ring is, thus, in equilibrium under classical electromagnetic forces alone, for the ring dimension *R*, as well as for the thickness dimension *r*. The spinning ring is completely stable because the forces upon it are in equilibrium, and its charge and current distributions do not vary with time. According to Maxwell theory, no radiation is possible; and without any means of losing energy, the ring retains its electromagnetic energy and shape.

THE NATURAL FREQUENCY AND SIZE OF THE RING

Still another way to see that the speed of the ring *c* is actually merely the velocity of an electromagnetic field is to consider the spinning ring as an *LC*-circuit. The resonant frequency of an *LC*-circuit with a standing electromagnetic field of velocity *c* is

$$\omega_{LC} = \frac{2\pi}{\sqrt{LC}} \tag{25}$$

* Weber's electrodynamics includes a force from acceleration which is entirely missing in the Maxwell theory.

From equations (5) and (6) this gives

$$\omega_{LC} = \frac{2\pi c}{\sqrt{4\pi^2 R^2}} = \frac{c}{R} \tag{26}$$

The Compton frequency is often associated with an electron. It is given by the Planck frequency condition where the energy is taken as the rest energy of the electron; thus,

$$\omega_C = \frac{mc^2}{\hbar} \tag{27}$$

Equating the natural resonant *LC*-frequency of the model ω_{LC}, equation (26), with the Compton frequency ω_C, equation (27), yields

$$\omega_{LC} = \omega_C \qquad\qquad R = \frac{\hbar}{mc} \tag{28}$$

The radius *R* of the ring equals the rationalized Compton wavelength, 3.86×10^{-13} meters. Previous estimates of the electron size have been both larger and smaller.* Quantum theory is generally assumed to specify a large smeared out distribution of mass and charge which can be on the order of atomic dimensions—about 10^{-10} meters. A vastly smaller estimate of $\sim 10^{-22}$ meters was recently published;[11] this is even smaller than the size of a proton. Yet the electron's magnetic moment is known to exceed the proton's, and equation (29) requires a larger electron size for reasonable estimates of charge rotations. Modern atomic models assign small protons to the nucleus and larger electrons to the volume surrounding the nucleus.

THE MAGNETIC MOMENT OF THE ELECTRON

The magnetic moment of the model to the first approximation, using equation (4) is given by

$$\mu_e \equiv A'I = \frac{\pi R^2 e\omega}{2\pi} = \frac{ceR}{2} \tag{29}$$

where A' is the area of the current loop $A' = \pi R^2$. From equation (28) for *R* as the Compton wavelength, the model yields the magnetic moment to the first approximation as

$$\mu_e = \frac{\hbar e}{2m} \equiv \mu_B \tag{30}$$

where μ_B is the Bohr magneton. This fits the observed value to about 3 or 4 places. A closer approximation that includes the so-called anomalous magnetic moment is derived below.

ELECTRON SPIN

Evans states[12] "Empirically, it was necessary to assume that each electron possesses an intrinsic angular momentum, in addition to its usual orbital angular momentum, as though it

* On the basis of scattering experiments, Compton estimated the radius of the ring electron is about 2×10^{-12} m. [A. H. Compton, "The Size and Shape of the Electron," Phys. Rev. Sec. Series **14**, 247-259 (1919).]

were a spinning rigid body. The observable magnitude of this spin angular momentum is $\hbar/2$." This magnitude of the angular momentum of spin

$$p_s = \frac{\hbar}{2} \tag{31}$$

should be identified with the *free* electron and should not be confused with the spin of an electron bound in an atom and coupled to other atomic particles.*

The angular momentum of a free electron depends upon its magnetostatic energy only and not upon its electrostatic energy, since only *moving* charge will produce momentum. A charged non-spinning ring would have electrostatic energy, but no angular momentum. The spinning ring has the tangential velocity c, so that the spin angular momentum is

$$p_s = m_m cR \tag{32}$$

where

$$m_m = \frac{E_m}{c^2} \approx \frac{E_t}{2c^2} \approx \frac{m}{2} \tag{33}$$

is the mass equivalent of the magnetostatic energy, as given by equations (3) and (7).

The fact that only magnetostatic mass m_m is involved and the magnetostatic mass is one-half the total mass of the electron means that the angular momentum of the electron p_s, as given by equation (32), is only one-half the amount that would be associated with an ordinary spinning macroscopic body. This spinning ring model then explains why the electron has a gyromagnetic ratio μ_e/p_s twice as large as the classical ratio $e/2m$, or from equations (30) and (31)

$$\frac{\mu_e}{p_s} = 2\frac{e}{2m} \tag{34}$$

From equations (7), (28), (32), and (33), the spin of the electron according to the proposed model becomes

$$p_s = \frac{e^2}{8\pi^2 \varepsilon_o c} \ln\,(8R/r) = \frac{\hbar}{2} \tag{35}$$

If $R = \hbar/mc$ is the Compton wavelength, then the half-thickness r is to be chosen such that

$$\ln(8R/r) = \pi/\alpha \tag{36}$$

where $\alpha = e^2/4\pi\varepsilon_o c\hbar$ is the fine structure constant, and

$$r = 8R\,\exp(-\pi/\alpha) \approx 0 \tag{37}$$

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* For an atomic electron, "The total angular momentum of a single particle is the summation of its spin momenta, and is represented by the intrinsically positive number j. The magnitude of the angular momentum of the corresponding motion is $\hbar\sqrt{j\,(j+1)}$ ...[and] j is restricted to half-integer values of j = 1/2...."[12] Thus, although quantum theory specifies $p = \sqrt{3}\,\hbar/2$ for the combined orbital and spin momenta of atomic electrons, this value should *not* be taken as the spin momentum of the *free* electron.

The value of r is so small that it can be ignored except where a singularity might otherwise arise.

ANOMALOUS MAGNETIC MOMENT

To the first approximation, the spinning ring model of the electron yields one Bohr magneton for the magnetic moment of the electron, equation (30), in agreement with observations to 3 or 4 places. A more accurate expression is obtained by using a better value for the self inductance L' of the spinning ring. Instead of the second of equations (6), a more accurate expression for the self inductance is

$$L' = \mu_o R' \left[\ln(8R'/r') - 2 \right] \approx \mu_o R' \ln(8R'/r')(1 - 2\alpha/\pi) \tag{38}$$

where the approximation involves setting $\ln(8R'/r') = \pi/\alpha$ as given by equation (36) and $\alpha = e^2/4\pi\varepsilon_o c\hbar \approx 1/137$, and where the radius of the ring R' and the half-thickness r' are to be determined anew.[*]

The more accurate expression (38) reveals that the magnetostatic energy is slightly less than the electrostatic energy, leading to more accurate ring parameters. The relevant relations that must be satisfied by appropriate choices of the parameters R', r', and ω' are:

angular frequency:
$$\omega' = c'/R' \tag{39}$$

current:
$$I' = e\omega'/2\pi = ec'/2\pi R' \tag{40}$$

magnetostatic energy:
$$\begin{aligned} E'_m &= L' I'^2/2 \\ &= \left(e^2/8\pi^2\varepsilon_o R'\right)\left(c'/c\right)^2 \ln\left(8R'/r'\right)\left(1 - 2\alpha/\pi\right) \end{aligned} \tag{41}$$

electrostatic energy:
$$E'_e = \left(e^2/8\pi^2\varepsilon_o R'\right) \ln\left(8R'/r'\right) \tag{42}$$

angular momentum:
$$p'_s = m'_m \qquad R' = E'_m c' R'/c^2 = \hbar/2 \tag{43}$$

total energy:
$$E_t = E'_e + E'_m = mc^2 \tag{44}$$

magnetic moment:
$$\mu' = \pi R'^2 I' = \left(ceR'/2\right)\left(c'/c\right) \tag{45}$$

From Lenz' law, it may be assumed that the physical system will adjust so that changes in the parameters from the first approximation will be as small as possible. It may be assumed that the magnitude of the change in magnetostatic energy will equal the magnitude of the change in the electrostatic energy from symmetry; thus,

$$\left|E'_e - E_e\right| = \left|E'_m - E_m\right| \tag{46}$$

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[*] It is no longer possible to have both equipartition of energy between electrostatic and magnetostatic energy and to also have the required tangential velocity of the ring c' equal to c. The LC-frequency ω_{LC} is no longer applicable, as it is derived assuming equipartiton between magnetostatic and electrostatic energies. The Compton frequency ω_C, being related to no observation, cannot be used. The more accurate expression (38) for the magnetostatic energy is the important property leading to more accurate ring parameters.

Using these conditions, the radius of the spinning ring R need not be changed from the first approximation; thus,

$$R' = R = \frac{\hbar}{mc} \tag{47}$$

Only the angular frequency ω' and the half-thickness r' need to be chosen anew such that

$$\omega' = \omega(1+\alpha/2\pi) = (mc^2/\hbar)(1+\alpha/2\pi)$$

$$\ln(8R/r') = \ln(8R/r)(1+\alpha/2\pi) = \pi/\alpha + 1/2 \tag{48}$$

The new half-thickness r' then becomes

$$r' = \frac{8\hbar}{mc}\exp\left(\frac{-\pi}{\alpha} - \frac{1}{2}\right) \approx 0 \tag{49}$$

From equation (39), the new tangential velocity of the ring c' becomes greater than the former rim velocity c thus,

$$c' = c(1+\alpha/2\pi) \tag{50}$$

The new tangential velocity of the ring is *exactly* equal to the speed of light which may be shown by calculating the condition for dimensional stability of the ring using the exact equation for inductance. Equation (50) indicates that the first approximation of tangential velocity (labeled c) is actually slightly less than the speed of light. Substituting equations (47), (48) and (50) into (41) and (42), the new magnetostatic and electrostatic energies become

$$E'_m = E_m(1-\alpha/2\pi) \qquad E'_e = E_e(1+\alpha/2\pi) \tag{51}$$

which are seen to conserve energy from equations (7) and (44) and also the symmetry condition (46). Substituting the first of equations (51) and (47) into (43), it is seen that the angular momentum of spin p_S is conserved as $\hbar/2$. Substituting equations (47) and (50) into (45), the more exact magnetic moment becomes

$$\mu'_e = \mu_e(1+\alpha/2\pi) = (e\hbar/2m)(1+\alpha/2\pi) \tag{52}$$

The anomalous magnetic moment is then given by

$$\mu'_e/\mu_e - 1 = \alpha/2\pi = 0.0011613... \tag{53}$$

which agrees with the observations to 6 or 7 places. Considering the gyromagnetic ratio μ'_e/p'_S from equations (43) and (52), it is seen to possess the same anomaly $\alpha/2\pi$ as the magnetic moment.

Higher order approximations for the magnetic moment of the electron can be obtained by considering still more accurate expressions for the capacitance and self-inductance of the spinning ring.

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